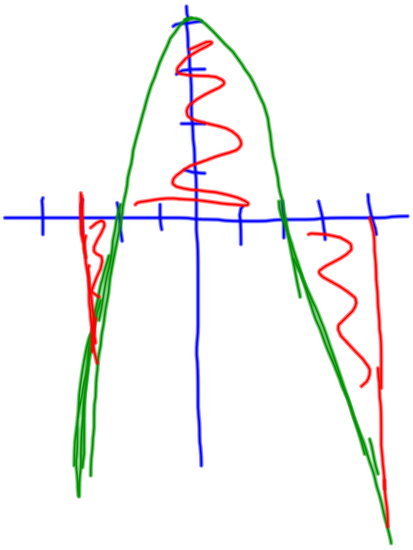
6.5/3 $f(x) = 4 - x^2$ $a = -3,\ b = 4$
$n = 4$

$\Delta x_1 = 1,\ \Delta x_2 = 2$
$\Delta x_3 = 1,\ \Delta x_4 = 3$
$x_1^* = -\frac{5}{2},\ x_2^* = -1,\ x_3^* = \frac{1}{4},\ x_4^* = 3$



Interval: $[-3, 4]$

Partition: $[-3,-2], [-2,0], [0,1], [1,4]$

$h = f(-\frac{5}{2})$ $h = f(-1)$ $h = f(\frac{1}{4})$ $h = f(3)$
$= -\frac{9}{4}$ 3 $\frac{63}{16}$ -5

$$\sum_{k=1}^{4} \Delta x_k f(x_k^*) = (1)\left(-\frac{9}{4}\right) + (2)(3) + 1\left(\frac{63}{16}\right) + (3)(-5)$$

$\approx -\frac{117}{16}$ (bob guess)

mesh size = $\max(\Delta x_k) = 3$

6.5 / 5

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (x_k^*)^2 \, \Delta x_k$$

$a=-1, b=2$

$(x_k^*)^2$ → height (a function value)

Δx_k → width

So the function could be - $f(x) = x^2$

$$\int_{-1}^{2} x^2 \, dx$$

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6.5 /9 9a)

$$\int_1^2 2x\,dx$$

$$= \lim_{\max \Delta x_k \to 0} \sum \Delta x_k \, 2(x_k^*)$$

6.5/8

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \left(\sin^2(x_k^*)\right) \Delta x_k \quad ; a = 0, b = \frac{\pi}{2}$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x) \, dx$$

Indefinite Integral

aka Antiderivative

$$\int f(x)\,dx = F(x) + C$$

whenever

$$\frac{d}{dx}\left(F(x)+C\right) = f(x)$$

This is a family of functions.

Definite Integral

limit of sums of generalized areas of rectangles

i.e. limit of sums of products of widths of subintervals and heights of functions.

This is a NUMBER

Also

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

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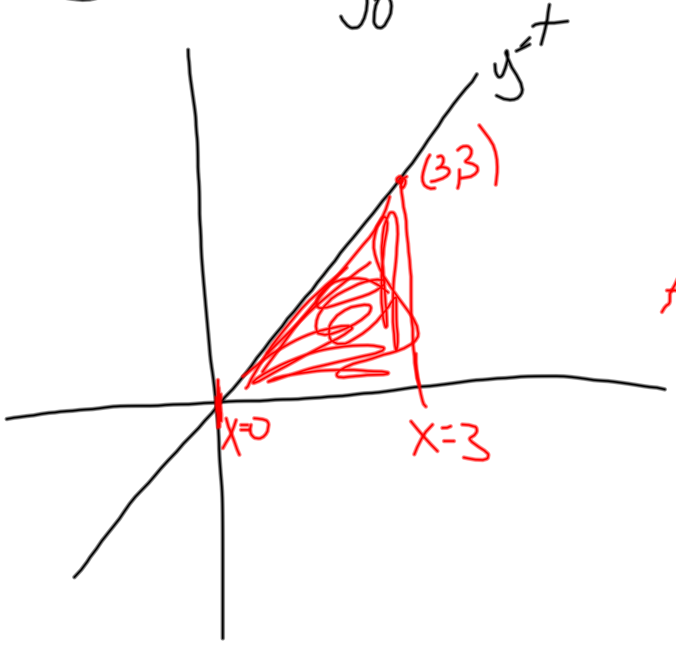
6.5/11

a) $\int_0^3 x\,dx$

i) sketch the region

ii) use the pic to evaluate the integral



$A_\Delta = \frac{1}{2}bh = b\left(\frac{1}{2}h\right)$

$= (3)\left(\frac{1}{2}(3)\right) = \frac{9}{2}$

14d

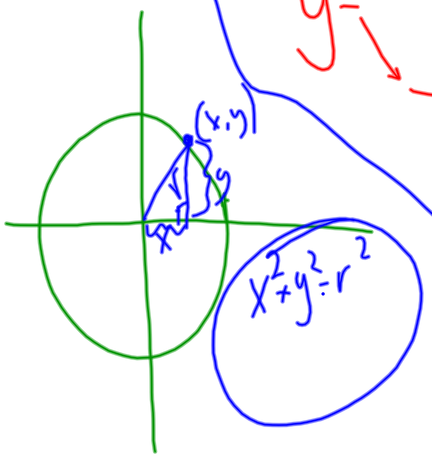
$x^2+y^2=r^2$ is the equation of a CIRCLE, with radius r, centered at the origin

$\quad -x^2 \qquad -x^2$

$\boxed{y^2=r^2-x^2}$

$y = +\sqrt{r^2-x^2}$ eqn of the TOP semicircle

$y = -\sqrt{r^2-x^2}$ " " " BOTTOM semicircle



Barrons
a # 38

$$\lim_{x\to 0} \frac{\cos x - 1}{x}$$

$$= \lim_{h\to 0} \frac{\cos(h) - 1}{h} = \lim_{h\to 0} \frac{\cos(0+h) - \cos(0)}{h}$$

$f(x) = \cos(x)$

$a = 0$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$f'(x) = -\sin(x)$

$f'(0) = -\sin(0) = 0$

barrons 49

$$\lim_{x\to 0} \frac{\tan(\pi x)}{x}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$\lim_{x\to 0} \frac{\sin(\pi x)}{x \cdot \cos(\pi x)}$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin(\pi x)}{\pi x}}_{\downarrow\ 1} \cdot \underbrace{\frac{\pi}{\cos(\pi x)}}_{\downarrow\ \frac{\pi}{1}}$$

$$= \pi$$

$$\lim_{h\to 0} \frac{\tan(\pi h)}{h} = \lim_{h\to 0} \frac{\tan(\pi h) - 0}{h}$$

$$= \lim_{h\to 0} \frac{\tan(\pi h) - \tan(0)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(\pi(0+h)) - \tan(\pi \cdot 0)}{h}$$

$$f(x) = \tan(\pi x)$$

$$a = 0$$

$$f'(x) = \sec^2(\pi x) \cdot \frac{d}{dx}(\pi x)$$

$$f'(x) = \pi \sec^2(\pi x)$$

$$f'(0) = \pi \sec^2(\pi \cdot 0) = \pi$$

Limit problem: what to do with "indeterminate forms"

1) $\frac{0}{0}$

2) $\frac{\infty}{\infty}$ $\lim_{x \to \infty} \frac{e^x}{x}$

3) 1^{∞} $\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^x$

4) $\infty - \infty$

5) ∞^0

6) 0^0

7) $0 \cdot \infty$

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1) Answer L'Hospital's Rule [L'Hôpital's]

If $\lim\limits_{x \to \left\{\begin{smallmatrix} a \\ \infty \\ -\infty \end{smallmatrix}\right\}} \frac{f(x)}{g(x)}$ exists and is of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$,

$$\lim_{x \to \{\}} \frac{f(x)}{g(x)} = \lim_{x \to \{\}} \frac{f'(x)}{g'(x)}$$

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$$\lim_{x\to 0} \frac{\cos x - 1}{x} = \lim_{x\to 0} \frac{-\sin x}{1} = 0$$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

$$\lim_{x\to 0} \frac{\sin(\pi x)}{x} = \lim_{x\to 0} \frac{\pi\cos(\pi x)}{1} = \pi$$

$$\lim_{x\to 0} \frac{2x^2+3x}{3x^2-700x} = \lim_{x\to 0} \frac{4x+3}{6x-700} = -\frac{3}{700}$$

$$\lim_{x\to \infty} \frac{2x^2+3x+1}{3x^2-700x+4} = \lim_{x\to \infty} \frac{4x+3}{6x-700} = \lim_{x\to \infty} \frac{4}{6}$$

$$= \frac{2}{3}$$